



Does more intelligent trading strategy win? Interacting trading strategies: an agent-based approach

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ABSTRACT: An artificial financial market is built on the top of Genoa Artificial Stock Market. The market is populated with agents having different trading strategies and they are let to interact with each other. Agents differ in the trading method they use to trade, and they are grouped as noise, technical, statistical analysis, and machine learning traders. The model is validated by replication of stylized fact in financial asset returns. We were able to replicate leptokurtic shape of probability density function, volatility clustering and absence of autocorrelation in asset returns. The wealth dynamics for each agent group is analysed throughout trading period. Agents with a higher time complexity trading strategy outperform those with strategy comparing their final wealth.

KEYWORDS: Agent based model, Multi-Agent Financial Market, ARIMA, Machine Learning

1. INTRODUCTION

The World Bank statistics reveal that the market capitalisation of all listed companies on stock exchanges in the world reaches a total of 94 trillion US dollars in 2020.¹ There have broad range of studies aiming to explain dynamics of asset prices and model this complicated financial market structures. However, the capital market theory for asset pricing and the efficient market hypothesis (EMH) assumption were the most common approaches used. These approaches assume that prices are efficiently valued, and individuals are homogenous and rational. However, these assumptions have been challenged by both empirical

data findings and complexity of the system. Therefore, alternative approaches have been introduced, Kahneman and Tversky (1979) proposed the prospect theory as a part of behavioural finance that describes how traders irrationally assess gain and losses asymmetrically. Cont (2001) also present a set of stylized facts of financial time series that cannot be explained by these traditional approaches. In this sense, agent-based models (ABMs) are introduced as a “paradigm shift” with more realistic assumptions as boundedly rational agents with heterogenous expectations. ABMs offer benefits over the traditional approaches such as emergent behaviour of system as result of interaction among system entities. Therefore,

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ABMs draw a wide attention and Jean-Claude Trichet, the former ECB president, writes that “We need to deal better with heterogeneity across agents and the interaction among those heterogeneous agents”.⁴

An ABM is a simulation to model a system consisting of interacting agents. Agents can have static or adaptive rules to initiate their interactions with other agents and environment. It has great importance in terms of providing bottom-up understanding of systems. However, it is very complicated to analytically model the interactions among market entities and agents can also apply range of sophisticated learning capabilities especially when continuous adaptation exists.⁵ In financial market perspective, traditional models fall short to explain the behaviour of market through extreme situations during financial crisis^{6,7} since there is no such classical approach to capture behaviour of crashing markets. In this sense, ABMs can capture such extreme moves when built with necessary components and optimal parameter calibrations.

Simulating stock markets has been growing field, many models are proposed focusing on market mechanism, wealth dynamics and price dynamics.^{9,10,11,12} The seminal paper of the Santa Fe Institute¹³ was pioneering work in this field alongside with several other financial market models: Genoa Artificial Stock Market.^{14,15,16} These models are differing in the way they set the market microstructure, agents trading strategies, network among agents and intelligence level in agents. A review of ABMs and its simulations in financial markets could be found in the literature.^{9,17,18,19} The main studies in this field propose that developing ABM requires a proper design and four main design elements are needed: market mechanism, trading strategies, traded assets and trader types. The built model is subject to be validated by measures of modelled market.

The validation is the key part of ABMs since it ensures the appropriateness of the simulation for the modelled system. The success of a financial market model is measured by the ability of reproducing stylized facts observed in the real market.³ Another approach for validation is to use modelled market parameters.^{20,21,22} Llacay and Peffer (2018) use face validation differing from the mainstream. The stylized facts in financial markets are used for validation in literature and they are *absence of linear autocorrelation*²³; *heavy tails*; *volatility clustering*^{24,25}; *volume/volatility correlation*³; *aggregational Gaussianity*. There is no simulation model can

reproduce all known facts due to increasing complexity of model, hence models are kept simple in compliance to Ockham’s razor principle which asserts to use minimal entity for explanations.

The trading strategies agents employ play a significant role in building a financial market simulation model.¹⁷ These strategies can range from zero-intelligent agents²⁶ to very intelligent agents compared to earlier studies.²⁷ In a recent study, Llacay and Peffer (2018) used agents with realistic trading strategies that takes historical price into account. The method used to take trade action mainly relies on future price forecast which can be any method, for example, evolutionary techniques such as genetic programming and artificial neural networks. Agents can also employ social learning method where agents observe other traders and change their strategy accordingly.^{9,28} However, this may lead a herding behaviour in the market in line with Hott (2009) study which shows the herding behaviour as a reason for bubbles in financial markets.

Considering main components of agent-based models for financial markets, trading methods are main agent diversifying component in the model. In this sense, considering existing studies, there are a few studies that takes realistic agent trading strategies since the earlier studies mainly employ agents with zero-intelligent and agents using fundamental value and genetic algorithms. In this study, we aim to fill this gap by including agents using more realistic technical and fundamental trading strategies as well as machine learning approaches. The methods our agents use have been studied in the literature for price prediction. For example, Ariyo et al. (2014) used Autoregressive Integrated Moving Average (ARIMA) and Nelson et al. (2017) used Long Short-Term Memory (LSTM) as predicting method. On the other hand, Llacay and Peffer (2018) applied some realistic technical trading tools in their agent-based model. However, the most of prediction methods use historical data and do back testing to measure the success of the model. Hence, they ignore the used method interaction with market environment, and this assumes no price impact in the market. Considering this fact, we equipped our agents with realistic trading strategies and let them to interact with all market entities. With this, the agent’s market effect is considered, and the model provides an insight into wealth dynamics of interacting agents. The model provides a realistic testbed for assessing financial

market hyper-parameters such as price tick size.

We extend the GASM model by adding interacting intelligent agents and analyse market dynamics and wealth dynamics. We aim to make four main contributions to the agent-based financial market modelling literature by: (1) reproduction and validation of the GASM model results; (2) use of more realistic trading strategies which are commonly used by practitioners (3) we analyse wealth dynamics of agent types hence, the effect of intelligence level on wealth return; (4) showing the catalyser role of noise traders in the market.

The rest of the paper is structured as follows: Section 2 presents our simulation model. In Section 4, simulation results are given. Section 5 discuss our findings and Section 6 concludes the study.

2. PROPOSED MODEL

The artificial financial market has similar microstructure with GASM model, for a detailed description of the model structure.⁴⁰ The herding behaviour phenomena is modelled different from GASM model. Agents form a cluster with given probabilities and the final cluster is activated with a given probability that all agents belong to the cluster are either seller or buyer.

2.1 Trader Types

Traders are engaged to buy and sell financial assets in financial markets for themselves or on behalf of another parties. Traders vary in perceiving the market, they therefore employ different strategies for trading. At this point, the market theories come into account and help traders to see different beliefs about these complex systems. There are several studies give evidence to either for or against EMH. For example, Fama (1965) finds that historical prices cannot predict the future prices while Brock et al. (1992) and Kwon and Kish (2002) evidence that technical trading rules can beat buy-and-hold strategy for DJIA, FT30 and NYSE stock markets, respectively. In addition to this, statistical methods such as ARIMA³⁰ and LSTM³¹ are used to predict future stock price for trading. In this sense, an environment with different types of agents reflects the heterogeneity of traders in real market. The literature in testing trading methods usually take a strategy as a baseline and do back testing to compare performances. Therefore, agent-based

approach fills this gap partially although it is not possible to mimic the entire complex real market dynamics.

In the light given facts, the artificial stock market is populated with six types of agents who are named as the method they are equipped with: Noise, Moving Average Convergence Divergence (MACD), Relative Strength Index (RSI), Bollinger Bands, ARIMA and LSTM. Agents will be named with the method they use in the rest of this paper. For all agents, the amount of assets (cash) to be traded is random fraction of assets(cash) and the limit price is a draw from a interval that is attached to historical volatility. Agents rely on their signal function when taking trading decision.

The noise traders have a great importance in keeping the market working since they act as a catalyser in the market and supply volume for intelligent traders.^{8,35} *The RSI* is considered as a momentum indicator that gives signal of overbought or oversold. The method is developed by Wilder (1978) and the RSI value range from 0 to 100 and the RSI value is regarded as overbought if it is above 70 while it is oversold when it is below 30. *The MACD* is a technical trader tool developed by Gerald Appel in late 1970s. It is mainly based on exponential moving average (EMA) which is a type of moving average that takes the more recent data points the greater weight. *The Bollinger Bands* is a technical trader tool developed by John Bollinger in 1980s. It is volatility measure indicator that relies on the past price of asset and its volatility. The agents using *ARMA*(p, q) as defined as in Tsay (2005), the multiple steps forecast with ARMA model is computed recursively. The ARIMA model use integrated data by differencing the raw data to meet the time series stationary. The ARIMA traders checks stationarity of stock price and do differencing till obtain a stationary series. The traders estimate ARIMA models with different lags to find optimal p and q values. They finally select the model with minimum Akaike information criterion (AIC). The forecast price values are predicted and that is fed into a decision-making process. *The LSTM* is recurrent neural network architecture developed by Hochreiter and Schmidhuber (1997). It is a machine learning method with deep networks and differs from feedforward neural networks with feedback connections since it can process sequences of data. The LSTM is widely used in predicting stock price movement and outperform baseline approaches.^{31,39} The LSTM traders use simulation initialisation period stock price return

to predict following 5-periods return so post orders accordingly.

3. MARKET INITIALISATION

At the beginning of simulation, the stock price p_0 is set to be \$100. The wealth is equally distributed among agents, each get 1000 stock (inventory) and \$100000 cash. The hyperparameters for market is set before simulation run as in Table 1.

There are total of 550 agent population of which 500 noise traders and 10 for each of RSI, MACD, Bollinger, ARIMA and LSTM traders. The tick size for asset price is one cent. Marchesi et al. (2003) extended the GASM model by populating the market with four different agents. Like this study, the most of agents are noise traders that enables the order matching mechanism working. The simulation time steps refer a trading day and simulation is consist of 5040 days which is approximately

20-year trading period since a year has average 252 trading days (Fig. 1).

Agents are in a partially observable environment since they only can access asset price. Agent types use technical trading indicators, statistical model for time series, and a machine learning, deep learning. All intelligent agents are reflective agents, rule-initiated, since they rely on signals for the forecast period. The stock market is closed form since there is no cash inflow.

The total wealth of agent i^{th} agent at time step t can be calculated as $w_i = c_i^t + a_i^t * p_t$, where c_i^t and a_i^t are the cash amount and assets of i^{th} agent at time step t and p_t asset price. The calculation of traders' final wealth is likewise. The wealth of a trader changes throughout simulation as a result of their interactions. The actions within market environment are based on the strategy trader employ to take buy or sell action. Building these strategies rely on the parameters that emulate realistic trading strategies, which is given in Table 2.

Table 1. Market initial parameters.

Market Parameters	Value	Description
N	550	Total number of agents
T	5240	Simulation time steps
PAC	0.001	Probability that agents create a cluster
PCA	0.002	Probability that cluster is activated
BP	0.5	Buy probability of noise traders
SMu	1.01	Mean of sell limit orders
SSK	4.5	Sell sigma K
BMu	1.01	Mean of buy limit orders
BSK	4.5	Buy sigma K
Agent Population	[500, 10, 10, 10, 10, 10]	Vector of agent population [Noise, RSI, MACD, Bollinger, ARIMA, LSTM]

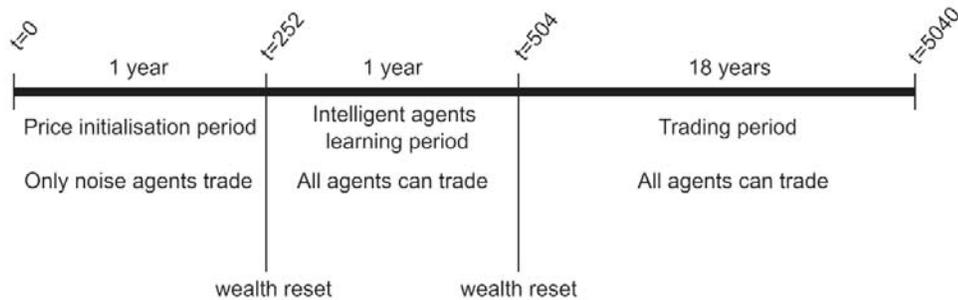


Figure 1. The artificial stock market periods.

Table 2. Agent initial parameters.

Agent Type	Parameters	Values	Description
Noise	$[p]$	[0,5]	Buy probability
RSI	$[n_r, v_p, v_u]$	[14, 30, 70]	Periods of RSI, buy signal threshold, sell signal threshold
MACD	$[n_{m1}, n_{m2}, n_{m3}, \alpha]$	[12, 26, 9, $2/(n+1)$]	EMA(p), EMA(p), EMA(MACD) periods and smoothing constant
Bollinger	$[n_b, k_b]$	[20, 2]	Periods and constant k
ARIMA	$[p, d, q]$	[[1], [0,1], [1]]	p, d, q are lag order of AR, degree of differencing and MA window size, respectively. p, q take an integer value 1 and 2, depending on model selection AIC criteria. d is mainly 0 or 1.
LSTM	[HiddenLayer, Optimiser, Epochs, LearningRate]	[20, "adam", 50, 1, 0.005]	HiddenLayer is the number of layers in between input and output layers. "Adam" is an optimiser for training deep neural networks. Epochs is the number of learning algorithm works through the training set. LearningRate is the step size of gradient descent on finding minimum of loss function.

2. SIMULATION MODEL AND RESULTS

In this section, the extended GASM model is simulated, and the result of the experiments are presented. We first let only noise traders to trade in the market for a given initialisation period hence, initial stock price is generated. The market then is populated with five more different traders who are called "intelligent" agents since those agents predict future price move. The market behaviour emerges under agent interactions.

The simulation is run with 500 noise traders and 10 intelligent traders for each method. Since the amount of asset to trade is a random friction of agent's wealth, having 10 agents for each method will decrease the effect of randomness on average. Several simulations with same parameters were run and all give similar outputs. Therefore, results here are a representative simulation model for those series of simulation. The flow of simulation model is given in Fig. 2.

The model keeps the GASM main structure, however, some parameters are tuned after several experiments and intelligent agents are added to the market. The population share of traders in the market are determined with experiments. A market with more than 10% of intelligent agent population leads stock

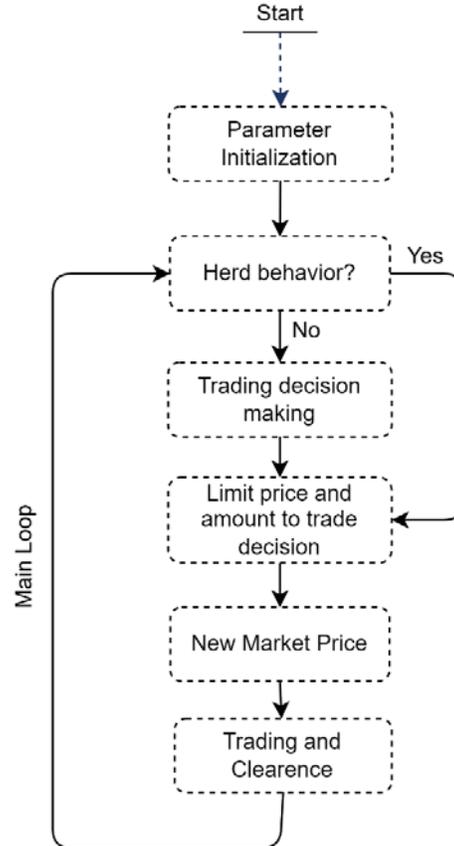


Figure 2. Stock Market Simulation Loop Structure.

Table 3. Agent decision estimation window and decision making.

Traders	Estimation Period (day(s))	Forecast Period (day(s))	Buy Rule (If ...)	Sell Rule (If ...)
Noise	-	-	$a > 0.5$, where $a \in U(0,1)$	$a < 0.5$, where $a \in U(0,1)$
RSI	14	1	$RSI < 30$	$RSI > 70$
MACD	9, 12, 26	1	$\Delta_t * \Delta_{t-1} < 0$ & $\Delta_t > 0$	$\Delta_t * \Delta_{t-1} < 0$ & $\Delta_t < 0$
Bollinger Bands	20	1	$p_t > SMA_{t,20}(pp) + 2 \cdot \sigma_p$	$p_t > SMA_{t,20}(pp) - 2 \cdot \sigma_p$
ARIMA	t	5	$\hat{p}_{t+1} > p_t$	$\hat{p}_{t+1} > p_t$
LSTM	t	5	$\hat{p}_{t+1} > p_t$	$\hat{p}_{t+1} > p_t$

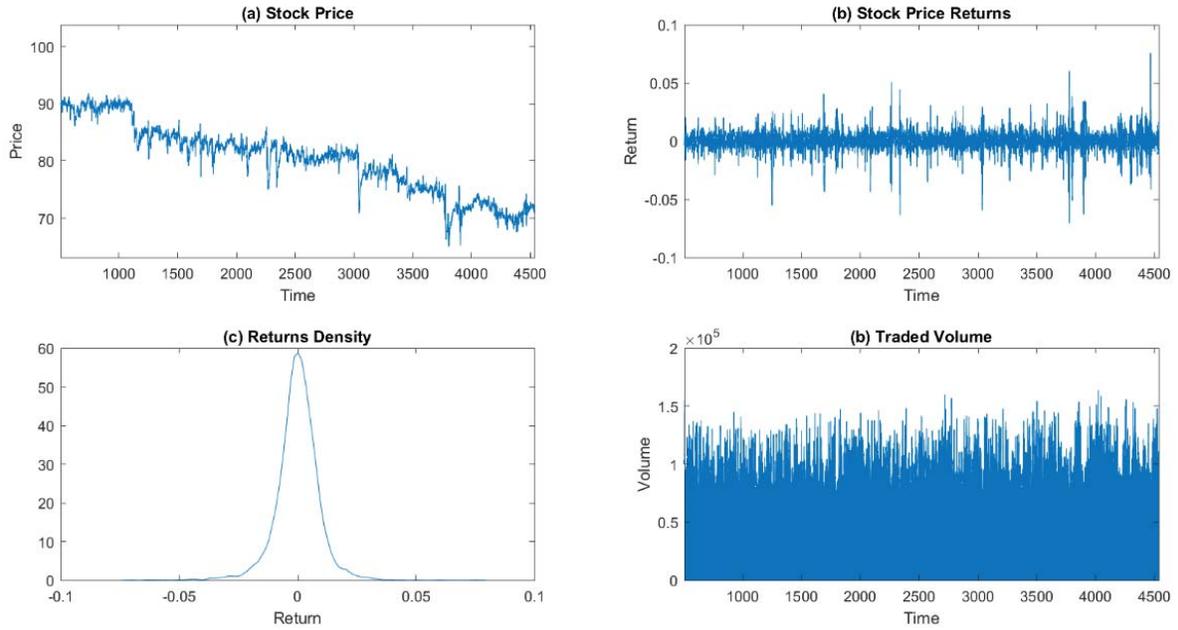


Figure 3. Market outputs over simulation time. Upper left panel: Asset price. Upper right panel: asset price log return. Lower left panel: asset price log return density distribution. Lower right panel : traded volume.

price jumps and halt in price formation process. The decision-making process is two part which are trading decision and the amount to trade. The amount to trade is random fraction of cash or assets. However, the trading decision depends on the method agents use, trading signal functions is summarised in Table 3. It shows the tuning options on parameters for agent trading methods hence, mostly used realistic trading parameters are used to condition realistic trading strategies.

2.1 Price, return and volume analysis

Financial market modelling with traditional approaches have assumption that stock returns are normally distributed. However, empirical findings show that returns have fatter tails than normal distribution.³ In addition to this findings, daily stock returns have some characteristics that are well documented in Warner and Brown (1985). Therefore, the price and other emergent features of simulated market

are supposed to exhibit these characteristics alongside stylized facts.

The price, return and volume outputs are presented in Fig. 3 and Table 4, the log returns of price is expressed as returns in the rest of this paper.

Table 4. Asset price and return related descriptive statistics.

Statistics	Price	Returns
Mean	81.98	0.000
Standard Deviation	6.58	0.0091
Minimum	65.12	-0.0702
Maximum	101.80	0.0754
Skewness	-0.33	-0.5172
Kurtosis	2.09	9.5849
Augmented Dickey-Fuller Unit Root Test	-0.6579	-87.2366***

Descriptives of price returns are in line with real world stock return features which has zero mean and have heavy-tailed distribution. The distribution is leptokurtic and left skewed with 11.65 kurtosis and -0.769 skewness measure. The price is not-stationary at level based on Augmented Dickey-Fuller test; it is first degree integrated series. The simulation parameters are tuned for different combinations of market and agent parameters. The most striking result is that increasing population of intelligent agents halts price formation so the market.

2.2 Validation

The validation of an agent-based financial market model is measured with the number of stylized facts the simulation model is capable to reproduce. The validity of our built model is tested by eligibility of outputs to real financial market features. As a seminal work, Cont (2001) documented a list of stylized facts for asset returns. Agent based models for financial markets have reproduce some these stylized facts but not all of them, so do ours. In addition to all market microstructure parameters, there are also six different types of agents interacting which increase the complexity of the stock market. The validation process is conducted for each fact given in Cont (2001).

Return autocorrelations

It is empirically showed that autocorrelation in asset returns is insignificant but intraday time scales could be exception.³ There would be a price to be exploited otherwise, and this is an assumption of efficient market hypothesis. The estimated autocorrelation coefficient function (acf) values for simulation generated asset price returns indicates that there is a statistically significant negative autocorrelations for first two lags and fast decaying values afterwards. This is more like intraday small time scales feature of asset returns (Fig. 4a). The slow decay behaviour in absolute return autocorrelation function is another real market feature as stated in Cont (2001) (Fig. 5a). Volatility clustering is another real market feature that is measured by squared returns

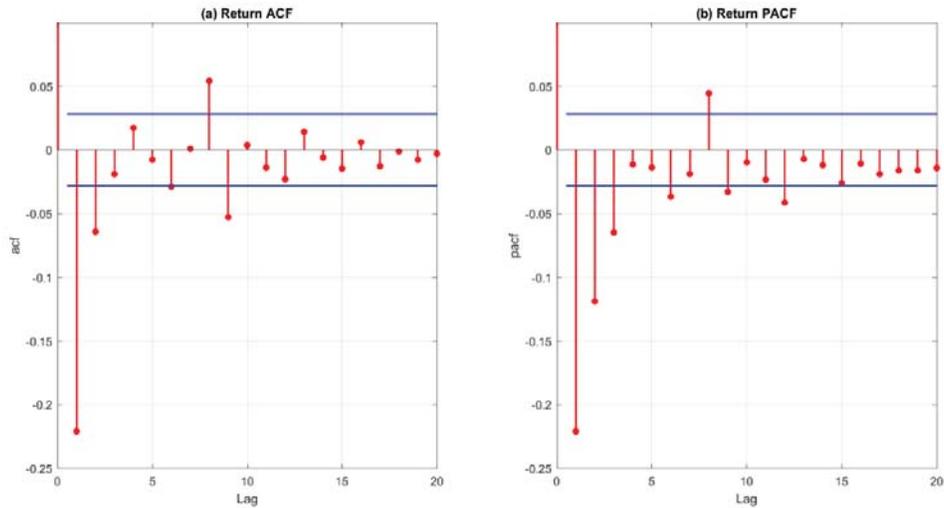


Figure 4. Return related autocorrelations. Left panel: return autocorrelation function. Right panel: Return partial autocorrelation function.

Note: Blue lines stands for 95% confidence interval.

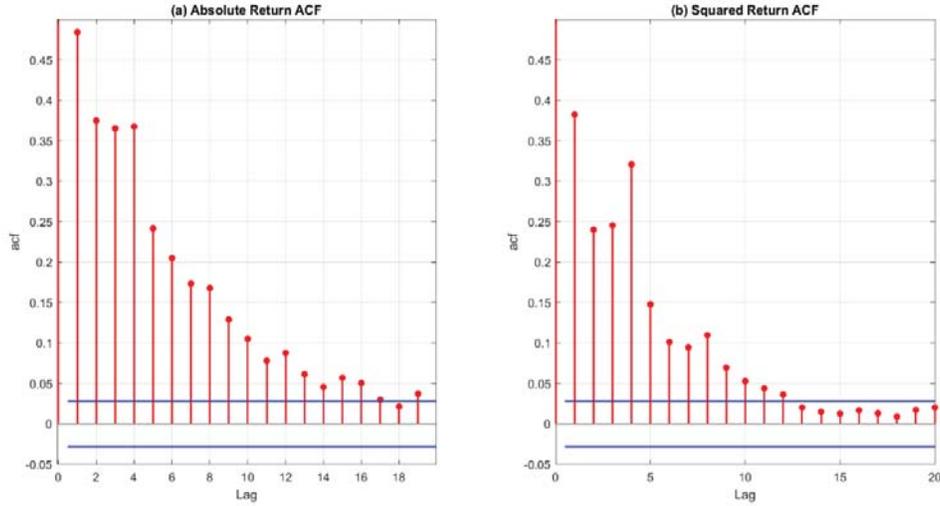


Figure 5. Volatility related autocorrelations. Left panel: absolute return autocorrelation function. Right panel: Squared return partial autocorrelation function.

Note: Blue lines stands for 95% confidence interval.

Table 5. List of stylized facts for asset returns that is used for simulation model validations.

Stylized fact	Testing	Does our model meet?
Absence of return autocorrelations	Autocorrelation plot	Partially
Heavy tails	Histogram Kurtosis	Yes
Slow decay of autocorrelation in absolute returns	Autocorrelation plot	Yes
Volatility clustering	Squared return autocorrelation plot	Yes
Aggregational Gaussianity	Skewness and Kurtosis	No
Volume/volatility correlation	Corelation	No
Leverage effect	Corelation	Partially

autocorrelation function (Fig. 5b) and tested with Ljung-Box Q-test. The test results show that there is an autocorrelation in squared return with test values [critical values] of 1960.22 [11.07], 2155.86 [18.31] and 2176.27 [24.99] for lag 5,10 and 15, respectively. This is a sign of long dependence of volatile market conditions so the conditional volatility behaviour.

Volume/return correlations

It is expected to asset return has negative correlation with volume, however the simulation output short fall to meet this feature since the calculated correlation is $r = 0,03$. Another stylized fact is leverage effect which is defined as negative correlation between return and change in volatility. The simulation output was able to reproduce a weak *leverage effect* with $r = -0,089$. The validity of our model with stylized facts is summarised on Table 5.

Testing all stylized facts given in Cont (2001) for asset price and volume outputs from simulation show that the model can replicate real market features and they are summarized in Table 5.

2.3 Wealth analysis

The literature in testing trading strategy methods relies on back testing mostly where the agent is assumed to have no market impact on price. However, trading agents have effect on market dynamics since they interact with market participants. This study aims to create a stock market testbed where agent interaction is considered, hence variety of sensitivity analysis can be applied. Satisfying some real market stylized facts, the agent-based model is capable of generate real market features. Therefore, the market is populated with different types of

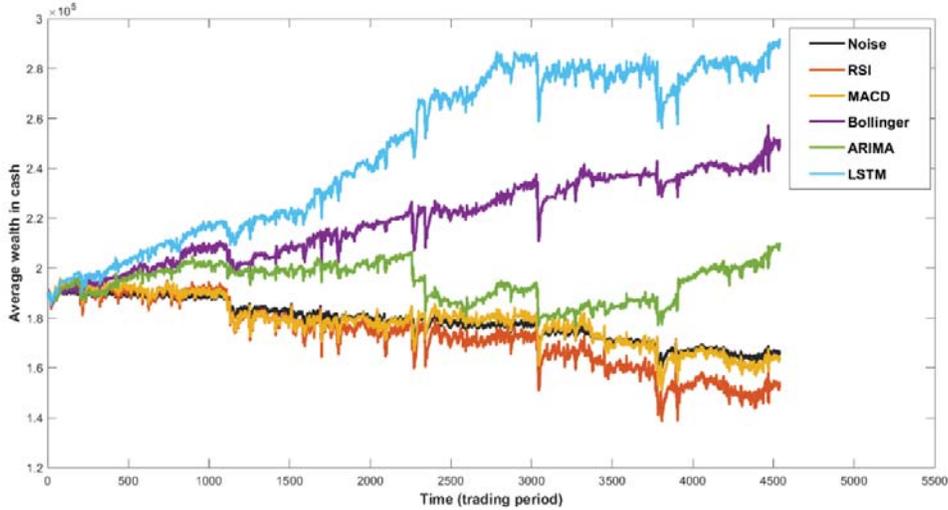


Figure 6. Average wealth of agent types in cash throughout trading period.

agents who compete to increase their wealth at the end of trading period.

One of the question this study aims to answer is if computationally intelligent agents can beat the overall market. In the light of this question, all agents behave as reflective agents with the signal they receive. The rules agent use to trade were summarised in Table 3. Based on these rules, agents entered market and start to trade. The average wealth of agent types over trading period is given in Fig. 6.

The agent named LSTM, which is a deep learning method, outperforms other agents by far. LSTM method is the most complicated and computationally costly method among others. Computation power can be considered as intelligence level in an interacting agent market. Therefore, it can be concluded that the more computational power the higher return. The number of days agents take long, and short positions is summarised in Table 5.

Table 5. Average number long and short positions over trading period.

Traders	Long positions	Short positions
Noise	2269	2266
RSI	120	131
MACD	368	369
Bollinger	125	126
ARIMA	127	4400
LSTM	2531	1811

Two agent group RSI and Bollinger are reluctant to take position since there is no up-down pattern in price long run. ARIMA and LSTM trade most of time since they take position based on their future price move prediction. The ANOVA is applied to mean wealth of agent types, agent wealth differs statistically at 1% significance level. The average wealth of agent type pairs was tested at 1%, except Noise-MACD agent pairs, the rest of 14 pairs has different wealth over the trading period. A boxplot for each agent group is created that also support this, see Fig. 7.

Although all agents belong to the same group use the same trading method, they differ in the amount to trade at each trading decision. Therefore, randomness in amount to trade decision give advantage to some traders. In this sense, each group has at least ten members and distribution checked at initial and final step to make sure same agent types are homogenous. To measure this, the Gini coefficient is calculated for all groups. The Gini coefficients is measure showing degree of inequality in wealth that ranges from 0 to 100. Zero coefficient means perfect equality while increase in it is a sign of inequality in wealth distribution. At the beginning of simulation all agents were endowed with same amount of wealth, hence the Gini coefficient was zero for each agent group. At the final timestep of simulation, the Gini coefficients are measured, and small inequalities occur during trading period since there is no coefficient greater than 10%, see Fig. 8.

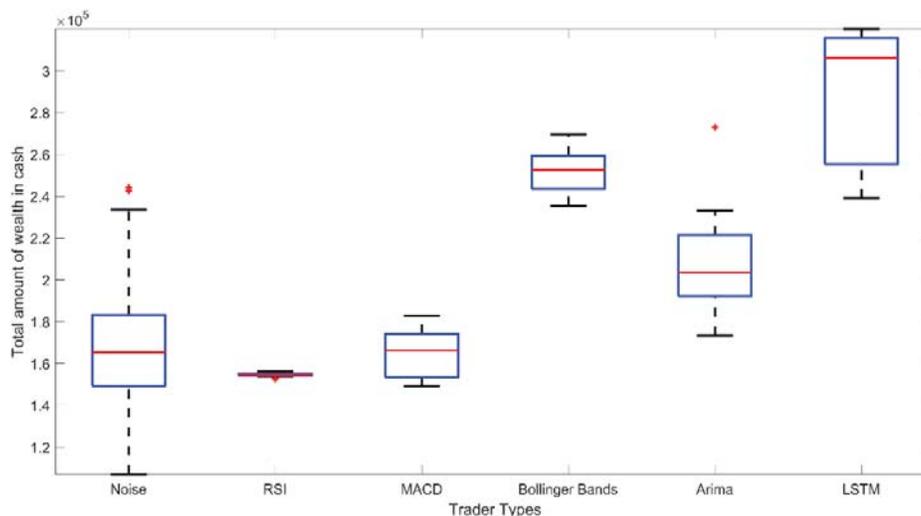


Figure 7. A boxplot for wealth comparison of different agent groups at final stage.

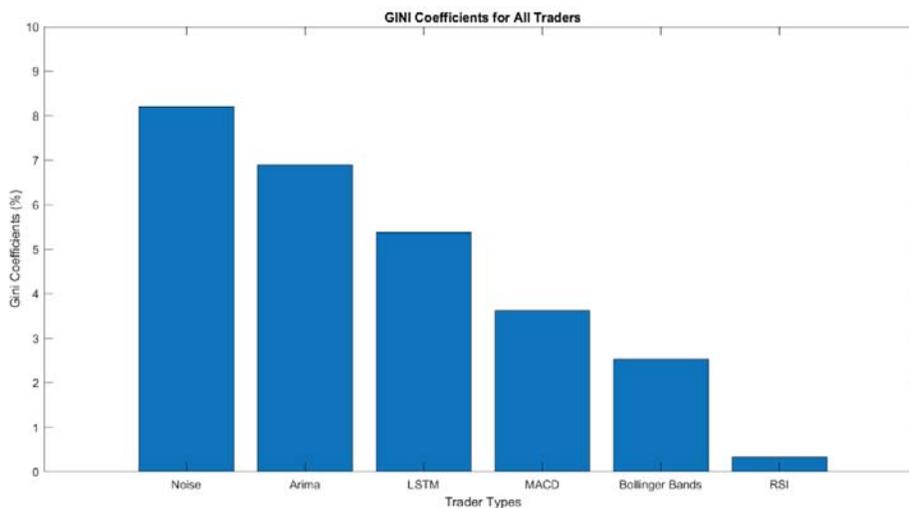


Figure 8. Gini coefficient of agent groups for total wealth at the final step of simulation.

Homogeneity in total wealth with same type agents were kept, and it remains stable at the end of simulation. Since the Gini coefficient is a measure of wealth inequality, the outliers in the wealth distributions lead higher coefficient that can be observed in Fig. 7 and Fig. 8.

3. DISCUSSION AND CONCLUSION

The study aims to gain a better understanding of trader interaction in stock markets and reproduce real market price features. An agent-based financial market simulation approach is employed to serve the purpose of this study since it takes agents' market impact

into account. The model was able to reproduce real market "stylized facts", thus it is eligible to be used as testbed for experiments. Hence, we were able to equip agents with realistic trading strategies. The findings provide both an insight into rivalry of different intelligence level in agents and supporting evidence to dominance of computationally powerful agents. It is evident that agent using deep learning approach get the highest return among others with the highest time complexity method.

The artificial stock market was populated with agent groups using no trading strategy, RSI, MACD, Bollinger, ARIMA and LSTM methods. Catalyser effect of noise traders is tested as the increase in population of

intelligent agents halts market and that is in line with Farmer et al., (2005). Zero intelligence in agents helps market to move and provide liquidity to the market. Our findings are also in line with back testing on real data, Siami-Namini et al. (2018) compares performance of ARIMA and LSTM methods where the LSTM trader outperforms. This is also can be taken as validity measure whereas Llacay and Peffer (2018) use also face validation and sensitivity analysis to validate their market model extended with realistic trading strategies.

Our results are consistent with the previous work of Raberto et al. (2001) and Marchesi et al. (2003) since it reproduces its results. Although it is challenging to represent complex dynamics of financial markets, a minimal model can still reproduce most of price dynamics.⁴³ The empirical findings of Cont (2001) fall into dispute with EMH assumptions. Therefore, a financial market with essential components is built and validity of empirical findings is tested. In addition to this, realistic trading strategies compete alongside agent interactions in our bottom-up market model. The emergent behaviour of the market is a result of agent interactions which is hardly traceable. Our agent-based financial market let agents to interact at micro level and analyse the behaviour of market dynamics under different parameter combinations. This can also be considered in a game theoretical view since competence of different strategies resulted in price equilibria. Considering these aspects, agent based financial market approaches can help us to better understand market dynamics even in a competing strategies environment.

There are potential limitations of study that heterogeneity in agents is more diverse in real markets such as informed and uninformed traders; high-frequency traders, value traders. Although our model mimic real market price features, fundamental value of an asset is the key for major investors and could be added as one trader type. A more powerful computation can ease time complexity of simulation when agents with complex trading strategy is considered such as deep learning method. The findings are obtained from a set of initial market parameters, different combination of parameters can be applied when modelling a specific market. This field also draw huge interest in high-frequency trading and limit order book modelling⁴⁴, therefore there are variety of direction to apply machine learning tools for future research.

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